Analysis of the Radiation Leakage for a Four-Aperture Phased-Array Applicator in Hyperthermia Therapy

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Abstract —We develop a criterion for the justified neglect of the field leaking from the regions between the horn apertures in a phased-array applicator. The condition is $(b/\delta)\psi \ll 1$, where δ is the skin depth of the conductive material, b is the radius of the torso or limb, and ψ is the wedge angle between the horn apertures.

I. Introduction

In view of the extensive use of phased-array-type applicators in hyperthermia, it is surprising that the relevant eletromagnetic boundary-value problems have not been adequately solved. Admittedly, the overall problem is vastly complicated and some idealizations and simplifications are desirable. It is important, however, to know what might be the consequences of making some gross assumption about the geometry. For example, in the case of phased-array applicators [1]–[4] arranged around the periphery of a cylindrical torso or limb, the apertures may have gaps between them. While we can specify the electric field within a given aperture with some assurance, it is certainly not obvious what the field is between the apertures. Using a two-dimensional model, we outline an analysis of this problem, which leads to a simple criterion for the neglect of the "radiation leakage." The proposed analytical method could also be employed to quantify some of these leakage effects, but we do not undertake this task here.

The situation is best described with reference to Fig. 1, where the idealized geometry is shown. Choosing cylindrical coordinates (ρ, ϕ, z), we define the target (i.e., limb, torso, or neck) as a homogeneous circular cylinder bonded by $\rho = b$. The conductivity, permittivity, and permeability are σ, ϵ, and μ_0, respectively. The fields are to be excited by four horn apertures with the electric field predominantly in the axial or z direction. In fact, we stipulate that

$$E_z(b,\phi) = F(\phi) \text{ for } \begin{cases} 0 < |\phi| < \alpha \\ \dfrac{\pi}{2} - \alpha < |\phi| < \dfrac{\pi}{2} + \alpha \\ \pi - \alpha < |\phi| < \pi \end{cases}. \quad (1)$$

where $F(\phi)$ is a known function over the angular width of each aperture. In fact, we restrict attention here to even-type excitation such that $F(\phi) = F(-\phi)$, but we wish to retain the freedom to introduce asymmetry about $\phi = \pi/2$. As indicated, the angular width of each aperture is 2α.

Now, in the case where $\alpha < \pi/4$, the electric field $E_z(b, \phi)$ cannot be assumed to be zero over the interval of ϕ between the edges of the apertures (e.g., when $\alpha < \phi < \pi/2 - \alpha$) unless, of course, we filled in these portions of the cylindrical surface with a metallic surface. When such a cast is not present, there is no reason why we should set $E_z(b, z) = 0$ over the gaps between the apertures. But one possible way to cope with this problem is to say that $E_z(b, \phi)$ and the corresponding magnetic field $H_\phi(b, \phi)$ are related by some type of impedance. There is a good reason to say this impedance will be substantially the same parameter Z_0 for each aperture. Thus, in effect, we are asserting that

$$E_z(b,\phi) \cong -Z_0 H_\phi(b,\phi) \text{ for } \begin{cases} \alpha < |\phi| < \dfrac{\pi}{2} - \alpha \\ \dfrac{\pi}{2} + \alpha < |\phi| < \pi - \alpha \end{cases}. \quad (2)$$

Turning our attention to the interior problem (i.e., $\rho < b$), we can write immediately the form of the solution for the axial electric field

$$E_z = \sum_{m=-\infty}^{+\infty} A_m I_m(\gamma\rho) e^{-im\phi} \quad (3)$$

where $I_m(\gamma\rho)$ is a modified Bessel function of order m and argument $\gamma\rho$ where $\gamma = [i\mu_0\omega(\sigma + i\epsilon\omega)]^{1/2}$ is the propagation constant for a time factor $\exp(i\omega t)$. Now the magnetic fields are obtained from

$$i\mu_0\omega H_\rho = -\partial E_z / \rho\,\partial\phi \quad (4)$$

and

$$i\mu_0\omega H_\phi = \partial E_z / \partial\rho \quad (5)$$

for $\rho < b$. In particular

$$H_\phi = y \sum_{m=-\infty}^{+\infty} A_m I_m'(\gamma\rho) e^{-im\phi} \quad (6)$$

where

$$y = \gamma/(i\mu_0\omega) = [(\sigma + i\epsilon\omega)/i\mu_0\omega]^{1/2}$$

and $I_m'(\gamma\rho)$ is the derivative of I_m with respect to $\gamma\rho$.

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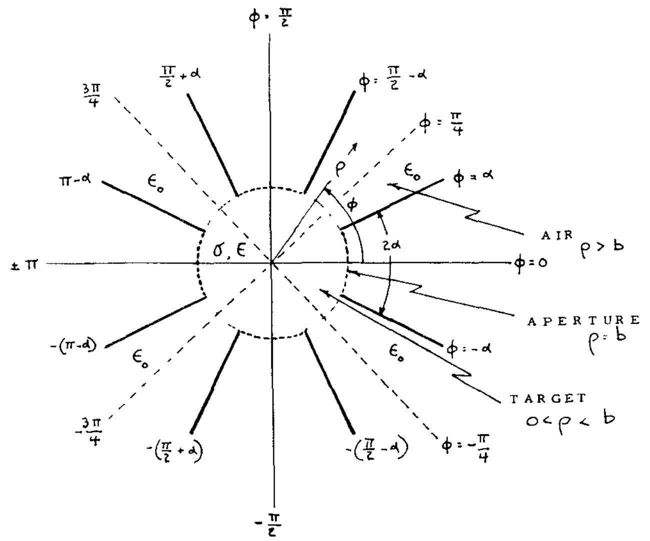

Fig. 1. Idealized two-dimensional model of a four-aperture phased-array applicator for hyperthermia, where we allow for the intervening air gaps between the horn apertures.

We now match the interior fields of the target with the surface fields at $\rho = b$, so that

$$\sum_{m=-\infty}^{+\infty} A_m I_m(\gamma b) e^{-im\phi} = \begin{cases} F(\phi), & \text{over apertures} \\ -Z_0 y \sum_{m=-\infty}^{+\infty} A_m I_m'(\gamma b) e^{-im\phi}, & \text{over gaps} \end{cases} \tag{7}$$

Both sides of this equation are now multiplied by $e^{in\phi}$ and the results are integrated over ϕ from $-\pi$ to π. Thus, we find that

$$2\pi A_n I_n(\gamma b) = \int_{\text{apertures}} F(\phi) e^{in\phi} d\phi - Z_0 y \sum_{m=-\infty}^{+\infty} A_m I_m'(\gamma b) \int_{\text{gaps}} e^{-i(m-n)\phi} d\phi. \tag{8}$$

Then, bearing in mind that $F(\phi)$ is an even function of ϕ, we see that

$$\begin{aligned} 2\pi A_n I_n(\gamma b) &= 2\int_0^{\alpha} F(\phi)\cos n\phi d\phi + 2\int_{\pi/2-\alpha}^{\pi/2+\alpha} F(\phi)\cos n\phi d\phi \\ &+ 2\int_{\pi-\alpha}^{\pi} F(\phi)\cos n\phi d\phi - Z_0 y \sum_{m=-\infty}^{+\infty} A_m I_m'(\gamma b) \\ &\cdot \left[2\int_{\alpha}^{\pi/2-\alpha} \cos[(m-n)\phi]\, d\phi + 2\int_{\pi/2+\alpha}^{\pi-\alpha} \cos[(m-n)\phi]\, d\phi \right]. \end{aligned} \tag{9}$$

Now we may assume that the distribution over each aperture is the same except for a complex constant. Thus, if we designate

$$2\int_0^{\alpha} F(\phi)\cos n\phi\, d\phi = f_n \tag{10}$$

then

$$\int_{\pi/2-\alpha}^{\pi/2+\alpha} F(\phi)\cos n\phi d\phi = f_n P \cos n\frac{\pi}{2} \tag{11}$$

and

$$2\int_{\pi/2-\alpha}^{\pi} F(\phi)\cos n\phi\, d\phi = f_n Q \cos n\pi \tag{12}$$

where P and Q are complex factors that can be adjusted in amplitude and phase to achieve a measure of focusing [3]. Also, we note that the integrations over the cosine functions in (9) are elementary. Thus, we deduce that

$$\begin{aligned} 2\pi A_n I_n(\gamma b) &= 2\left[1 + 2P\cos n\frac{\pi}{2} + Q\cos n\pi\right] f_n \\ &- Z_0 y \sum_{m=-\infty}^{+\infty} A_m I_m'(\gamma b) \frac{8}{m-n} \\ &\cdot \sin\left[\left(\frac{m-n}{2}\right)\left(\frac{\pi}{2} - 2\alpha\right)\right] \cos\left[\frac{\pi}{2}\left(\frac{m-n}{2}\right)\right]. \end{aligned} \tag{13}$$

Presumably (13) can be employed to get the desired coefficients A_n. A possible procedure is to truncate the series over m and then solve the resulting series of linear equations. In some cases, a simple perturbation procedure should suffice; we merely neglect the series over m and solve for A_n. This value is then inserted into the summand in (9) to yield a first-order corrected value for A_n. The process can be repeated as desired. The convergence should be best for small angular gaps $(\pi/2 - 2\alpha)$ between the apertures and/or if Z_0 is sufficiently small.

The value of Z_0 to employ in the calculation is not obvious. But one possible approach is to regard the gap regions between the apertures as wedge-shaped sectors that are bounded by radial planes (e.g., $\phi = \alpha$ and $\phi = \pi/2 - \alpha$). A suitable solution for the axial electrical field, within the first sector (i.e., for $\alpha < \phi < \pi/2 - \alpha$), is of the form

$$E_z = \sum_q a_q H_{\nu_q}^{(2)}(k\rho) \sin\left[\nu_q(\phi - \alpha)\right] \tag{14}$$

which is the radially outgoing solution of the Helmholtz equation $(\nabla^2 + k^2)E_z = 0$, where $k^2 = \epsilon_0 \mu_0 \omega^2 = \omega^2/c^2$ and $H^{(2)}$ is the Hankel function of the second kind of argument $k\rho$ and order ν_q. On requiring that E_z vanish at $\phi = \alpha$ and $\pi/2 - \alpha$, we deduce that

$$\nu_q = \frac{q\pi}{(\pi/2) - 2\alpha}, \qquad \text{where } q = 1, 2, 3, \cdots.$$

The coefficients a_q are not known at this stage. The corresponding form of the azimuthal component of the magnetic field is given by

$$\begin{aligned} H_\phi &= \frac{1}{i\mu_0\omega} \frac{\partial E_z}{\partial \rho} \\ &= -\frac{i}{\eta_0} \sum_q a_q H_{\nu_q}^{(2)\prime}(k\rho) \sin\left[\nu_q(\phi - \alpha)\right] \end{aligned} \tag{15}$$

where $\eta_0 = (\mu_0/\epsilon_0)^{1/2} = 120\pi$ and the prime indicates differentiation with respect to $k\rho$. The fields in the second sector (i.e., for $\pi/2 + \alpha < \phi < \pi - \alpha$) are identical in form to (14) and (15) if $\phi-\alpha$ is replaced by $\phi-(\pi/2+\alpha)$ and the coefficients a_q are replaced by a new set, say b_q $(\neq a_q)$.

Now it is possible to employ the general form given by (14) and (15) to generate a sequence of boundary-matching equations at the cylindrical interface $\rho=b$. We will argue that only the leading terms (i.e., for $q=1$) need be retained. This provides for a sinusoidal variation of the fields across the gap between the apertures. Thus, in the region $\alpha<\phi<\pi/2-\alpha$, we obtain

$$Z_0=-\frac{E_z}{H_\phi}\bigg|_{\rho=b}=-i\eta_0\frac{H_{\nu_1}^{(2)}(kb)}{H_{\nu_1}^{(2)\prime}(kb)} \tag{16}$$

where $\nu_1=\pi/(\pi/2-2\alpha)$.

In the (unlikely) limiting case where $kb\gg\nu_1$, we see that $Z_0\to\eta_0$. However, if $kb\ll\nu_1$, we find that

$$Z_0\simeq i\eta_0\frac{kb}{\nu_1}=i\mu_0\omega b/\nu_1=\frac{i\mu_0\omega b}{\pi}\left(\frac{\pi}{2}-2\alpha\right). \tag{17}$$

While it appears that $|Z_0|$ will be small compared with η_0 (or $120\pi\Omega$), the key dimensionless parameter in (13) is the product

$$Z_0y=\frac{i\mu_0\omega b}{\pi}\left(\frac{\pi}{2}-2\alpha\right)\left(\frac{\sigma+i\epsilon\omega}{i\mu_0\omega}\right)^{1/2}$$
$$=\gamma b\left(\frac{\pi}{2}-2\alpha\right)/\pi. \tag{18}$$

Thus, we can say that the condition for the justified neglect of radiation "leakage" from the system is that

$$(b/\delta)\psi\ll 1$$

where $\delta=1/\text{Re}\cdot\gamma$ is the skin depth of the conductive material and $\psi=(\pi/2)-2\alpha$ is the wedge angle between the horn apertures.

In the discussion above, we have not specified or assumed the form of the aperture illumination at each of the horns. A reasonable supposition [4] is to choose $F(\phi)=E_0\cos((\pi/2\alpha)\phi)$ for $-\alpha<\phi<\alpha$. Then it is a simple matter to deduce from (10) that

$$f_n=\frac{(\pi/\alpha)\cos n\alpha}{[\pi/(2\alpha)]^2-n^2}\qquad \text{if } n\alpha\neq\pi/2$$
$$=\alpha\qquad \text{if } n\alpha=\pi/2. \tag{19}$$

II. Conclusion

In summary, we can state that the criterion for neglecting the leakage radiation should be valid for typical applicators. Although the assumptions are reasonable, further analyses and confirming experimental data would be useful.

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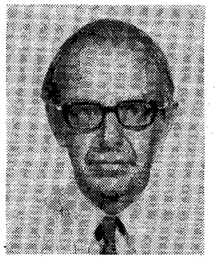

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